

personal watercraft, forty-seven cars (including five Hummers, four Ferraris, three Dodge Vipers, two DeTomaso Panteras, and a Lamborghini Diablo), two Rolex watches, a twenty-one-carat diamond bracelet, a \$200,000 samurai sword, and a commercial-grade cotton candy machine would have been difficult to explain as necessary business expenditures, Lawrence and his pals tried to cover their tracks by moving investors' money through a complex web of bank accounts and shell companies to give the appearance of a bustling and growing business. Unfortunately for them, a suspicious forensic accountant named Darrell Dorrell compiled a list of over 70,000 numbers representing their various checks and wire transfers and compared the distribution of digits with Benford's law. The numbers failed the test.<sup>3</sup> That, of course, was only the beginning of the investigation, but from there the saga unfolded predictably, ending the day before Thanksgiving 2003, when, flanked by his attorneys and clad in light blue prison garb, Kevin Lawrence was sentenced to twenty years without possibility of parole. The IRS has also studied Benford's law as a way to identify tax cheats. One researcher even applied the law to thirteen years of Bill Clinton's tax returns. They passed the test.<sup>4</sup>

Presumably neither the Harlem syndicate nor its customers noticed these regularities in their lottery numbers. But had people like Newcomb, Benford, or Hill played their lottery, in principle they could have used Benford's law to make favorable bets, earning a nice supplement to their scholar's salary.

In 1947, scientists at the Rand Corporation needed a large table of random digits for a more admirable purpose: to help find approximate solutions to certain mathematical equations employing a technique aptly named the Monte Carlo method. To generate the digits, they employed electronically generated noise, a kind of electronic roulette wheel. Is electronic noise random? That is a question as subtle as the definition of randomness itself.

In 1896 the American philosopher Charles Sanders Peirce wrote that a random sample is one "taken according to a precept or method which, being applied over and over again indefinitely, would in the long run result in the drawing of any one of a set of instances as often

as any other set of the same number."<sup>5</sup> That is called the frequency interpretation of randomness. The main alternative to it is called the subjective interpretation. Whereas in the frequency interpretation you judge a sample by the way it turned out, in the subjective interpretation you judge a sample by the way it is produced. According to the subjective interpretation, a number or set of numbers is considered random if we either don't know or cannot predict how the process that produces it will turn out.

The difference between the two interpretations is more nuanced than it may seem. For example, in a perfect world a throw of a die would be random by the first definition but not by the second, since all faces would be equally probable but we could (in a perfect world) employ our exact knowledge of the physical conditions and the laws of physics to determine before each throw exactly how the die will land. In the imperfect real world, however, a throw of a die is random according to the second definition but not the first. That's because, as Moshe pointed out, owing to its imperfections, a die will not land on each face with equal frequency; nevertheless, because of our limitations we have no prior knowledge about any face being favored over any other.

In order to decide whether their table was random, the Rand scientists subjected it to various tests. Upon closer inspection, their system was shown to have biases, just like Moshe's archetypally imperfect dice.<sup>6</sup> The Rand scientists made some refinements to their system but never managed to completely banish the regularities. As Moshe said, complete chaos is ironically a kind of perfection. Still, the Rand numbers proved random enough to be useful, and the company published them in 1955 under the catchy title *A Million Random Digits*.

In their research the Rand scientists ran into a roulette-wheel problem that had been discovered, in some abstract way, almost a century earlier by an Englishman named Joseph Jagger.<sup>7</sup> Jagger was an engineer and a mechanic in a cotton factory in Yorkshire, and so he had an intuitive feel for the capabilities—and the shortcomings—of machinery and one day in 1873 turned his intuition and fertile